M.Sc. (Mathematics) (New CBCS Pattern) Semester-IV

PSCMTH17 - Partial Differential Equations

P. Pages : 2 **GUG/S/25/13768**

Time : Three Hours Max. Marks : 100

Notes : 1. Solve all **five** questions.
2. All questions carry equal marks.

UNIT - I

1. a) Find the general integral of the P.D.E. $z(xp-yq)=y^2-x^2$. **10**

b) Prove that: The singular integral of $f(x,y,z,p,q)=0$ satisfies the following equations- **10**

$f(x,y,z,p,q)=0$

$f_p(x,y,z,p,q)=0$

$f_q(x,y,z,p,q)=0$

OR

c) Find the complete integral of $\left(p^2+q^2\right)y-qz=0$. **10**

d) Prove that a necessary and sufficient condition that the Pfaffian differential equation $\bar{X}\cdot d\bar{r}=P(x,y,z)dx+Q(x,y,z)dy+R(x,y,z)dz=0$ be integrable is that $\left(\bar{X}\cdot \text{curl}\,\bar{X}\right)=0$. **10**

UNIT - II

2. a) Prove that if an element (x_0,y_0,z_0,p_0,q_0) is common to both an integral surface $z=z(x,y)$ and a characteristics strip, then the corresponding characteristics curve lies completely on the surface. **10**

b) Find the characteristic stirps of the equation $xp+yq-pq=0$ and obtain the equation of the integral surface through the curve $C: z=x/2, y=0$. **10**

OR

c) Find the solution of $z=p^2-q^2$ which passes through the curve $c: x_0=s, y_0=0,\ z_0=\frac{-1}{4}s^2$. **10**

d) Find the integral surface of the p.d.e. $(x-y)y^2p+(y-x)x^2q=\left(x^2+y^2\right)z$ through the curve $xz=a^2, y=0$. **10**

UNIT - III

3. a) Reduce the equation $u_{xx} - x^2 u_{yy} = 0$ to canonical form. **10**

b) Drive the second order partial differential equation which describes the temperature distribution in a homogeneous isotropic solid. **10**

OR

c) Obtain D'Alembert's solution of the dimensional wave equation which describes the vibrations of a semi-infinite string. **10**

d) Prove that for the equation $Lu = u_{xy} + \frac{1}{4}u = 0$, the Riemann function is $\nu(x, y : \alpha, \beta) = J_0\sqrt{((x-\alpha)(y-\beta))}$, where J_0 denotes Bessel's function of the first kind of order zero. **10**

UNIT - IV

4. a) Show that the solution for the Dirichlet problem for a circle of radius α is given by Poisson integral formula. **10**

b) Find the condition that a one parameter family of surfaces form a family of equipotential surfaces. **10**

OR

c) Show that the solution of the Dirichlet problem is stable. **10**

d) State and prove the Harnack's theorem. **10**

5. a) Eliminate the parameters a and b from the equation $z = x + ax^2y^2 + b$ and find the corresponding partial differential equation. **5**

b) Discuss the method to find integral surface a semi-linear partial differential equation. **5**

c) State Green's Theorem. **5**

d) Discuss the Dirichlet problem and the Neumann problem. **5**

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